AP Calculus AB

Course Outline

I believe that using sound pedagogy and knowing individual student needs provide students the opportunity to be successful at mathematics. Setting expectations high will enhance student performance. Good communication of course objectives and requirements will inevitably lead to success.

Students will be required to communicate mathematics both in written and oral form, using both algebraic representation and written sentences.

Students are expected to come to class prepared and ready to participate both individually or as a member of a group. While I will be available before and after school for help sessions, peer tutoring is an integral part of being successful.

The course will be presented in a "rules of four" approach: using Numerical Analysis where data is known but not necessary an equation, Graphical Analysis where the graph but the equation is known, Algebraic Analysis using the traditional algebra and variable manipulation and verbal or written methods of representing problems.

Technology Requirement

Students will use a TI-84 Plus graphing calculator regularly in class and during study sessions. Calculators will be used on some but not all assessments. There is a need to be able to compute derivatives and indefinite integrals without the use of technology. There is a classroom set. Students who are unable to purchase their own calculator will be allowed to check out a graphing calculator for the school year.

Calculators will be used to conduct exploration; graph functions and analyze graphs; solve equations; justify and interpret results.

The classroom is equipped with a laptop lab and a SMART Board.

Unit 1: Pre-Calculus Review and Calculus Warm-up (1 week)

A. Functions
   1. Polynomials
   2. Composition of Functions
   3. Exponential and Logarithmic Functions
   4. Trigonometric Functions

Unit 2: Limits and Continuity (3–4 weeks) [C2]

A. Rates of Change
   1. Average Speed
   2. Instantaneous Speed

B. Limits at a Point
   1. 1-sided Limits
   2. 2-sided Limits
   3. Sandwich Theorem

      A Graphical Exploration is used to investigate the Sandwich Theorem. Students graph y1 = x^2, y2 = -x^2, y3 = sin (1/x) on graphing calculators. The $\lim_{x\to 0}$ of each function is explored. This investigation develops the visual idea of the Sandwich Theorem. The technique will aid students to "see" the $\lim_{x\to 0} x^2 \sin(\frac{1}{x})$. This helps tie the graphical implications of the Sandwich Theorem to the analytical applications of it. **[C3] [C5]

C. Limits involving infinity
   1. Asymptotic behavior (horizontal and vertical)
   2. End behavior models
   3. Properties of limits (algebraic analysis)
   4. Visualizing limits (graphic analysis)

D. Continuity
   1. Continuity at a point
   2. Continuous functions
   3. Discontinuous functions
      a. Removable discontinuity (0/0 form)

         **Groups conduct a tabular investigation

$$\lim_{x\to 1}(x^2 - 7x - 6)/(x - 1)$$

         Next, an analytic investigation of the same function is conducted. Students discuss with their groups any conclusions. Finally, a graphical investigation (using the graphing calculators) is conducted. Groups then report to the class their conjectures and discuss differences and similarities. **[C3] [C4] [C5]**

      b. Jump discontinuity
      c. Infinite discontinuity

E. Rates of Change and Tangent Lines
   1. Average rate of change
   2. Tangent line to a curve
   3. Slope of a curve (algebraically and graphically)

4. Normal line to a curve (algebraically and graphically)
5. Instantaneous rate of change

Unit 1 and 2 exam

Unit 3: The Derivative (5–6 weeks) [C2]

A. Derivative of a Function
1. Definition of the derivative (difference quotient)
2. Derivative at a Point
3. Relationships between the graphs of f and f'
4. Graphing a derivative from data

    A CBL experiment is conducted with students tossing a large ball into the air. Students graph the height of the ball versus the time the ball is in the air. The calculator is used to find a quadratic equation to model the motion of the ball over time. Average velocities are calculated over different time intervals and students are asked to approximate instantaneous velocity. The tabular data and the regression equation are both used in these calculations. These velocities are graphed versus time on the same graph as the height versus time graph. **[C3] [C5]
5. One-sided derivatives

B. Differentiability
1. Cases where f'(x) might fail to exist
2. Local linearity

    An exploration is conducted with the calculator in groups. Students will graph y1 = $|x|$ +1 and y2 = $\sqrt{(x^2 + 0.0001)} + 99$. They investigate the graphs near x = 0 by zooming in repeatedly. The students discuss the local linearity of each graph and whether each function appears to be differentiable at x = 0. **[C4] [C5]
3. Derivatives on the calculator (Numerical derivatives using "nDERIV")
4. Symmetric difference quotient
5. Relationship between differentiability and continuity
6. Intermediate Value Theorem for Derivatives

C. Rules for Differentiation
1. Constant, Power, Sum, Difference, Product, Quotient Rules
2. Higher order derivatives

D. Applications of the Derivative
1. Position, velocity, acceleration, and jerk
2. Particle motion

    **Application to physics
3. Economics

    a. Marginal cost

    b. Marginal revenue

    c. Marginal profit

    **Application to business

E. Derivatives of trigonometric functions

F. Chain Rule

G. Implicit Differentiation
1. Differential method
2. y' method

H. Derivatives of inverse trigonometric functions

I. Derivatives of Exponential and Logarithmic Functions
1. Logarithmic Differentiation
**Look at the proof of the Product and Quotient Rules for differentiation using Logarithmic Differentiation.

Unit 3 exam

Unit 4: Applications of the Derivative (5–6 weeks) [C2]

A. Extreme Values
1. Relative Extrema
2. Absolute Extrema
3. Extreme Value Theorem
4. Definition of a critical point

B. Implications of the Derivative
1. Rolle's Theorem
2. Mean Value Theorem
3. Increasing and decreasing functions

C. Connecting f' and f'' with the graph of f(x)
1. First derivative test for relative max/min
2. Second derivative
a. Concavity
b. Inflection points
c. Second derivative test for relative max/min

D. Optimization problems
**Living in a rural area, most of the students have constructed fencing for farm animals. This lab has students investigate how to optimize 100 feet of fencing for a rectangular pen using an existing wall of the school. Students must set up three different size pens. Groups must make a conjecture on the optimal size pen. Students then are asked to write up a model of a rectangular pen using the appropriate domain, find the optimal area using the derivative algorithm.
**Students are to work in partners to design an open-top box of optimal volume out of a piece of notebook paper. Each group is to report their ideas back to the class.

Unit Quiz

E. Linearization models
1. Local linearization

An exploration using the graphing calculator is conducted in table groups where students graph $f(x) = (x^2 + 0.0001)^{0.25} + 0.9$ around x = 0. Students algebraically find the equation of the line tangent to f(x) at x = 0. Students then repeatedly zoom in on the graph of f(x) at x = 0. Students are then asked to approximate f(0.1) using the tangent line and then calculate f(0.1) using the calculator. This is repeated for the same function, but different x values further and further away from x = 0. Students then individually write about and then discuss with their tablemates the use of the tangent line in approximating the value of the function near (and not so near) x = 0. **[C3] [C4] [C5]

2. Tangent line approximation
3. Differentials

Video: Newton's Dark Secrets, NOVA 2005

F. Related Rates

**Here Students participate is a "Tootsie Pop" lab. Students suck on a lollipop to determine the rate of change of their radius of their candy. They can calculate the candy's rate of volume change. Students measure the initial circumference with thread. They suck on the lollipop for thirty second intervals, recording the circumference. They calculate the appropriate radii and write a model with respect to time. Students can use this to estimate the rate of change of the volume of the lollipop when the radius is 75% of its original size. This lab is in its original form from the book A Watched Cup Never Cools published by Key Curriculum Press.

**Kite flying lab. Depending on weather conditions, students time the string let out while launching a kite. They use current wind speed. They can measure distance directly under the flying kite. Using differentiation with respect to time of the Pythagorean theorem, students can calculate the speed the kite is rising.

Unit quiz

Cumulative Exam

Unit 5: The Definite Integral (3–4 weeks) [C2]

A. Approximating areas

1. Riemann sums
   a. Left sums
   b. Right sums
   c. Midpoint sums
   d. Trapezoidal sums
2. Definite integrals

Students are asked to graph the identity function. They are asked to calculate a definite integral from x = -3 to x = 5 using varying number of partitions known geometric methods. Students then share their work with their group and are asked to come up with a table observation. Those observations are shared with other groups. The class discusses to make a conjecture about a formula. **[C3]

B. Properties of Definite Integrals

1. Power rule
2. Mean value theorem for definite integrals

An exploration is conducted to show students the geometry of the mean value theorem for definite integrals and how it is connected to the algebra of the theorem. **[C3]

C. The Fundamental Theorem of Calculus
1. Part 1
2. Part 2-Integral Evaluation Theorem

Unit exam

Unit 6: Differential Equations and Mathematical Modeling (4 weeks) [C2]

A. Slope Fields

B. Anti-derivatives
1. Indefinite integrals
2. Power formulas
3. Trigonometric formulas
4. Exponential and Logarithmic formulas

C. Separable Differential Equations
1. Growth and decay
2. General differential equations
3. Newton's law of cooling

Unit exam

Unit 7: Applications of Definite Integrals (3 weeks) [C2]

A. Integral as net change
1. Calculating distance traveled (particle motion)
2. Consumption over time
3. Net change from data

Video: Infinite Secrets: The Genius of Archimedes, NOVA 2003

B. Area between curves
1. Area between a curve and an axis
   a. Integrating with respect to x
   b. Integrating with respect to y
2. Area between intersecting curves
   a. Integrating with respect to x
   b. Integrating with respect to y

Unit quiz

C. Calculating volume
1. Cross sections
   **Use play-doh and plastic knives to slice cross sections
2. Disc method
   **Students have a quick paper cutting lab to make a paper bell table center. Visual helps student "see" rotation and solid formed.

3. Shell method (utilize staking cups as visual aid)

Unit quiz

Unit 8: Review/Test Preparation (3–5 weeks)

A. Multiple-choice practice (Items from past exams are used as well as items from review books.)
1. Test taking strategies are emphasized
2. Individual and group practice are both used

B. Free-response practice (Released items from the AP Central website are used liberally.)
1. Rubrics are reviewed so students see the need for complete answers
2. Students collaborate to formulate team responses
3. Individually written responses are crafted. Attention to full explanations is emphasized **[C4]**

Video: Proof, Fermat's Last Theorem

Unit 9: After the exam…

A. Projects designed to incorporate this year's learning in applied ways

B. Research writing projects on the historical development of mathematics and biography on mathematicians.

C. Advanced integration techniques
**I like to look at arc length using integration. I use a lab for fitting points to a hanging string length and computing the quadratic function that the string closely maps Then computing the integral using Trig Substitution technique to find the string length. Students then compare to actual string length and make conjectures on errors.

Textbook:

Finney, Demana, Waits and Kennedy. *Calculus—Graphical, Numerical, Algebraic*. Third edition. Pearson, Prentice Hall, 2007.

This textbook will be the class's primary resource. Students will benefit from reading it. It contains a number of interesting explorations that the class will conduct with the goal of students to be that of discovering fundamental calculus concepts. I will also explain topics in a way that students have found helpful over the years that I have been teaching calculus. I encourage cooperative learning, I believe the entire class benefits from working together and discussing questions and discoveries to help one another construct understanding. **[C4]**

Supplemental Texts:

Edwards and Penney. *Calculus with Analytic Geometry Early Transcendental*. Fifth edition. Prentice Hall, 1997.

Kelley. *The Humongous Book of Calculus Problems.* Penguin Group, 2006.

Mathematicians have been responsible for many great developments throughout history. Much of our understanding of the universe is a direct result of the contributions of mathematicians. Although you will find much accomplishment in doing mathematics, I hope you learn to view math as more than just numbers, variables, processes, and algorithms. I hope you learn to apply your mathematical understanding to help you create a better understanding of the Mathematical nature of our lives.